

# A quantum motor: directed wavepacket motion in an optical lattice

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We propose a method for arbitrary manipulations of a quantum wavepacket in an optical lattice by a suitable modulation of the lattice amplitude. A theoretical model allows to determine the modulation corresponding to a given wavepacket motion, so that arbitrary atomic trajectories can be generated. The method is immediately usable in state of the art experiments.

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The fine manipulation of wavepackets is a fundamental requirement in a large number of fields. Particularly important examples are quantum transport [1–5], “quantum simulators” that aim to reproduce solid state models [6–12], quantum information [13–16] and quantum metrology [17]. The possibility of trapping very cold atoms by light was a decisive step leading to enormous progress in this field. Tailored optical potentials, created by multiple interfering beams interacting with cold atoms, allow to trap and to guide atomic wavepackets for long times (compared to the dynamics of the atom external degrees of freedom) and distances (compared to the de Broglie wavelength). Moreover, using far off-resonance beams reduces decoherence effects to negligible levels. With such techniques old problems of quantum dynamics have been experimentally studied, as the elusive Bloch oscillations [18, 19] or quantum chaos [20–26]. In the emerging, and rapidly developing, field of quantum information, controlled motion in optical lattices provides ways to manipulate *q-bits* [27, 28]. The next step in the development of such techniques is the shaping and displacing of wavepackets at will, and this is the problem addressed in the present work. We consider the motion of a wavepacket in a driven two-dimensional (2D) optical lattice (the generalization to the 3D case is straightforward). By carefully engineering the temporal driving of the optical potential, we demonstrate a way to coherently impinge to an atom in an lattice almost any kind of motion, including coherent rotations of its wavepacket. Tailored wavepacket motions have been previously studied both theoretically and experimentally by using Bloch oscillations [12, 29], but in this case the amplitude of the motion is directly limited by the spatial extension of the wavepacket coherence. Controlled motion in modulated lattices has also been observed experimentally in [2, 12]. Our work shows how a suitably “overmodulated” driving allows to displace a wavepacket along paths of arbitrary shape with controlled (and even engineered) dispersion.

Let us first consider the quantum dynamics of an atom in a 1D tilted (or “washboard”) potential formed by a sinusoidal potential superposed to a constant force  $F_x$ , or a linear potential  $F_x x$ . We use normalized variables such that lengths are measured in units of the lattice step  $d$

( $= \lambda_L/2$ ,  $\lambda_L = 2\pi/k_L$  being the laser wavelength), energy in units of the recoil energy  $E_R = \hbar^2 k_L^2 / (2M)$  where  $M$  is the atom mass, and time is measured in units of  $\hbar/E_R$ . The corresponding Hamiltonian is

$$H_x = -\frac{1}{2m^*} \frac{\partial^2}{\partial x^2} + [V_x + A_x(t)] \cos(2\pi x) + F_x x, \quad (1)$$

where  $m^* = \pi^2/2$  is the mass in normalized units,  $F_x$  is the constant force measured in units of  $E_R/d$ ,  $\hbar = 1$ , and  $V_x$  is the lattice amplitude, to which a time-dependent component  $A_x(t)$  can be added. Tilted optical lattices have been experimentally realized by many groups [18, 19, 30].

Dynamics in a tilted lattice is conveniently described by *Wannier-Stark* (WS) states [19, 31–34], which are the eigenstates of the Hamiltonian of Eq. (1) with  $A_x(t) = 0$ . In numerical simulations a finite lattice is used, but if the bounds of the lattice are far from the region of interest there is no essential difference in the dynamics. The eigenenergies of the Hamiltonian. (1) form a “ladder” structure separated by the “Bloch frequency”  $\omega_B \equiv 2\pi/T_B = F_x$  ( $\omega_B = F_x d/\hbar$  in usual units). Depending on the ratio  $V_x/F_x$  each well can host more than one WS state, each family of states then forming its own ladder [32]. Throughout this paper the parameters of the potential and the initial conditions are chosen so that the atomic dynamics is accurately described by the *lowest-ladder* WS states only. This situation can be realized experimentally by using cold enough atoms and raising the optical potential adiabatically [19]. As WS states are, for the parameters used in the present work, highly localized on a potential well, we label a WS state by the well index  $n$  in which it is centered  $\varphi_n(x)$ . These states are invariant under a translation of a integer number  $n$  of lattice steps, provided the associated energy is also shifted by  $n\omega_B$ , that is

$$\varphi_n(x) = \varphi_0(x - n) \quad (2)$$

$$E_n = E_0 + n\omega_B \quad (3)$$

(in what follows we set  $E_0 = 0$ ). It is well known that a wavepacket submitted to the Hamiltonian of Eq. (1) with  $A_x(t) = 0$  has an oscillatory behavior, called Bloch oscillation, of period  $T_B$ . Adding a time-dependent potential

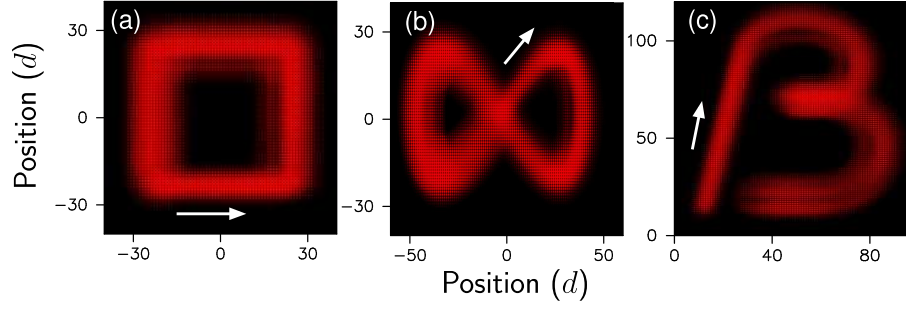


Figure 1: (Color online) Arbitrary wavepacket motion in two dimensions. The plots display the probability of presence  $|\Psi_{xy}(t)|^2$  integrated on time, and the arrows indicate the sens of the motion. (a) Square path formed of four straight lines. (b)  $\infty$ -path, obtained by choosing  $\alpha_x = \cos(\Omega t)$ ,  $\alpha_y = \cos(2\Omega t)$ ,  $\beta_x = \beta_y = 0$ . Plot (c) is made by combining straight lines and arcs. Parameters of the potential are  $V_{x,y} = 2.5$  and  $F_{x,y} = 0.2$  (note the different scales in each plot).

modulated at (or around) the frequency  $\omega_B$  is thus a good way to create a resonant response in the dynamics.

We can expand the atomic wave function  $\psi_x(t)$  over the WS states (of the first ladder):

$$\psi_x(t) = \sum_n c_n(t) e^{i\phi_n(t)} \varphi_n(x), \quad (4)$$

where the phase  $\phi_n(t)$  is conveniently defined by:

$$\phi_n(t) = -n\omega_B t - V_x M_0 \int_0^t A_x(t') dt' \quad (5)$$

and  $M_0$  is the  $p = 0$  value of the coupling parameter  $M_p$ :

$$M_p \equiv \langle \varphi_i | \cos(2\pi x) | \varphi_{i+p} \rangle = \langle \varphi_0 | \cos(2\pi x) | \varphi_p \rangle. \quad (6)$$

[the last identity is a consequence of Eq. (2)]. Bringing Eq. (4) into the Schrödinger equation associated to the Hamiltonian Eq. (1) one obtains the following set of equations:

$$\dot{c}_n(t) = -i A_x(t) \sum_{p \neq 0} M_p c_{p+n} e^{-ip\omega_B t}. \quad (7)$$

Eqs. (7) can be simplified by using the fact a WS state overlaps significantly only with the WS states associated with the nearest neighbor sites, that is  $M_p \approx 0$  for  $|p| > 1$ .

We now set in Eq. (1) the driving as

$$A_x(t) = \alpha_x(t) \sin(\omega_B t + \beta_x). \quad (8)$$

where  $\alpha_x(t)$  is a slowly varying function ( $|\alpha_x^{-1} d\alpha_x/dt| \ll \omega_B$ ). An additional simplification is obtained by neglecting fast oscillating terms; one then finds from Eq. (7):

$$\dot{c}_n(t) = \frac{M_1 \alpha_x(t)}{2} [c_{n-1} e^{-i\beta_x} - c_{n+1} e^{i\beta_x}] \quad (9)$$

whose general solution is

$$c_n(t) = \sum_p c_{n+p}(t=0) e^{ip\beta_x} J_p \left( -M_1 \int_0^t \alpha_x(t') dt' \right) \quad (10)$$

( $J_p(x)$  is the Bessel function of the first kind and order  $p$ ) which is a straightforward extension of a previous result [35]. The mean position of the wavepacket, averaged over a Bloch period  $T_B$ , is

$$\langle x \rangle_t = \langle x \rangle_{t=0} + \int_0^t v_x(t') dt',$$

where

$$v_x(t) = M_1 \alpha_x(t) \text{Re}(\sigma e^{-i\beta_x}) \quad (11)$$

is the instantaneous wavepacket drift velocity and  $\sigma \equiv \sum_p c_p^*(0) c_{p+1}(0)$  is the initial coherence of the wavepacket. The wavepacket also presents a diffusion characterized by an instantaneous diffusion coefficient  $D(t) \propto [M_1 \alpha_x(t) \text{Im}(\sigma e^{-i\beta_x})]$ . Hence, the motion and the diffusion can be controlled by changing the temporal driving  $\alpha_x$  and phase  $\beta_x$ . The diffusion can be suppressed by setting  $\beta_x$  so that  $\text{Im}(\sigma e^{-i\beta_x}) = 0$ ; one then obtains an undeformed translation of the wavepacket with a velocity given by Eq. (11),  $v_x(t) = \pm M_1 \alpha_x(t) |\sigma|$ . Conversely, setting  $\beta_x$  so that  $\text{Re}(\sigma e^{-i\beta_x}) = 0$  leads to a purely diffusive motion with no displacement of the wavepacket's center of mass.

Consider now a “square” 2D lattice formed by the interference of two orthogonal pairs of counterpropagating laser beams. It can be shown that the resulting Hamiltonian is separable [29]:  $H_{xy} = H_x + H_y$ , where  $H_x$  is given by Eq. (1) and  $H_y$  is the obvious generalization for the  $y$  coordinate. The solution can be written as  $\Psi_{xy}(t) = \psi_x(t) \psi_y(t)$  and the 2D dynamics is obtained simply by solving two identical 1D Schrödinger equations for each  $\psi_u(t)$  ( $u = x, y$ ), with the corresponding Hamiltonian  $H_u$ . We can thus induce a controlled 2D motion of the wavepacket by choosing suitable lattice modulations  $A_u(t) = \alpha_u(t) \sin(\omega_B t + \beta_u)$ .

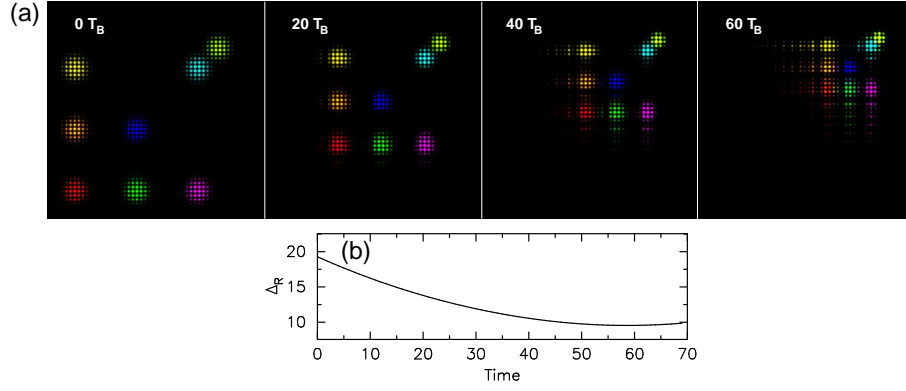


Figure 2: (Color online) The left plot on the top row displays a few wavepackets (or different slices of a large wavepacket) arbitrarily distributed in space. The potential defined by Eq. (13) sets the different wavepackets in motion with a velocity directed *towards* the origin (situated close to the upright corner) whose amplitude increases with the distance from the origin. The slices are thus progressively concentrated around the origin. Plot (b) shows the evolution of the total spatial dispersion with respect to the origin. Parameters of the potential are  $V_{x,y} = 2.5$ ,  $F_{x,y} = 0.25$  and  $k/k_L = 0.02$ .

In order to illustrate the possibilities opened by our method, we present in Fig. 1 different trajectories obtained by numerical integration of the Schrodinger equation with fixed parameters and an initial wavepacket of gaussian shape

$$\Psi_{xy}(0) = \sum_{l,m} \exp\left(-\frac{l^2 + m^2}{9}\right) \varphi_l(x) \varphi_m(y). \quad (12)$$

Note that this form implies that  $\sigma$  is real, the “zero diffusion” condition is thus fulfilled if  $\beta_x = \beta_y = 0$ . Plots (a)-(c) in Fig. 1 show the square-modulus of the wavepacket *integrated on time*, and the arrows indicate the sense of the motion. Plot (a) shows a square trajectory obtained with  $\alpha_x = 1$ ,  $\alpha_y = 0$ , for  $0 \leq t \leq 15T_B$ ,  $\alpha_x = 0$ ,  $\alpha_y = 1$  for  $15T_B \leq t \leq 30T_B$ , and so on. In plot (b), we drive the wavepacket into an  $\infty$ -shaped Lissajous curve by setting  $\alpha_x(t) = \cos(\Omega t)$ ,  $\beta_x = 0$  and  $\alpha_y(t) = \cos(2\Omega t)$ ,  $\beta_y = 0$ , with the overmodulation frequency  $\Omega = \omega_B/250$ . By combining paths one can generate any type of trajectories in 2D (and, by an obvious generalization of the above discussion, in 3D): Plot (c) in Fig. 1 displays a  $\beta$ -shaped trajectory, where even a turning point has been “drawn”. In all these numerical experiments the diffusion is very small, the width of the wavepacket varies only slightly during the evolution.

New kinds of dynamics can be obtained by using an overmodulation whose amplitude varies slowly *also in space*, i.e  $A_x(t) \rightarrow A_x(x,t)$  in Eq. (1). This can be realized [8, 36] by adding a second laser beam with a different spatial period  $k'_L$ , which produces a spatial modulation of the lattice amplitude corresponding to the beat note of the two spatial frequencies. The potential is then

$$A_u(u,t) = \sin(ku) (\sin \omega_B t + \beta_u) \quad (13)$$

where  $k = (k_L - k'_L)/k_L$ . In the limit  $|k| \ll 1$ ,  $\sin(ku) \approx ku$  and the velocity of the wavepacket, which is propor-

tional to the modulation amplitude, varies linearly with the *position*, as one can deduce from Eq. (11):  $v_u = M_1 \sigma k u$  (taking  $\beta_u = 0$ ). All parts of the wavepacket, whatever their positions, will move towards the origin  $(0,0)$ , and the closer of the origin it is, the smallest its velocity. This effect can thus be used to concentrate a large wavepacket, or various wavepackets distributed at arbitrary positions, on a few potential wells. Fig. 2(a) shows a numerical experiment in which such a concentration is realized. Each colored spot represents a wavepacket, or a part of a large wavepacket. As times goes by, the different spots are seen to converge to the same position. Plot (b) in Fig. 2 shows the evolution of the total spatial dispersion  $(\langle \Delta r^2 \rangle^{1/2})$ . This unavoidable dispersion limits the maximum time during which the overmodulation can be applied, and thus the maximum density which can be obtained.

One can also combine temporal and spatial modulations to produce a *rotation* of a wavepacket. Consider a cigar-shape wavepacket as shown in Fig. 3(a). Inducing a uniform rotation around the origin  $(0,0)$  means to give a slice at position  $\mathbf{r}$  a local velocity  $\mathbf{v}$  perpendicular to  $\mathbf{r}$ . This can be done by producing a modulation of the form

$$\begin{aligned} A_x &= -\sin(ky) \sin(\omega_B t + \beta_x) \approx -ky \sin(\omega_B t + \beta_x) \\ A_y &= \sin(kx) \sin(\omega_B t + \beta_y) \approx kx \sin(\omega_B t + \beta_y). \end{aligned}$$

Figure 3 shows an example of wavepacket rotation produced by this technique.

In conclusion, this paper illustrates the almost unlimited power of overmodulated optical lattices to manipulate atom wavepackets. The technique is immediately applicable to state-of-art experiments, and that can play an important role in preparing complex initial state for fundamental studies of quantum dynamics, but might also be useful for playing with atomic qubits in the realm

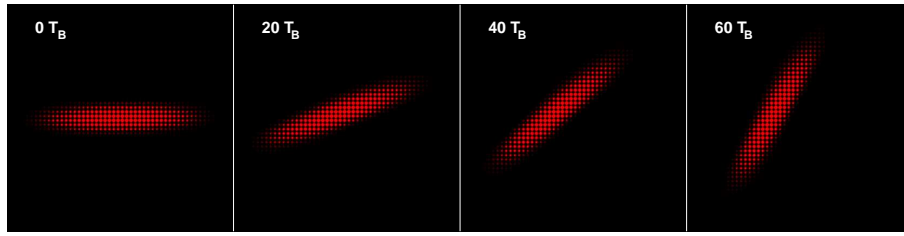


Figure 3: (Color online) Rotation of the wavepacket, obtained by using combined spatial and temporal overmodulations (cf. text).

of quantum information experiments. Although we have discussed only 2D examples, that are easier to understand and to display pictorially, all the techniques illustrated above are straightforwardly generalized to the 3D case.

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